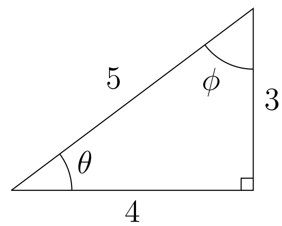
Inverse Trig Practice- 9/30/16

1. What is the sine, cosine, and tangent of θ and of ϕ in the following picture?



Solution: $\sin(\theta) = 3/5$, $\cos(\theta) = 4/5$, $\tan(\theta) = 3/4$, $\sin(\phi) = 4/5$, $\cos(\phi) = 3/5$, $\tan(\phi) = 4/3$.

2. $\arccos(\sqrt{3}/2) = ?$

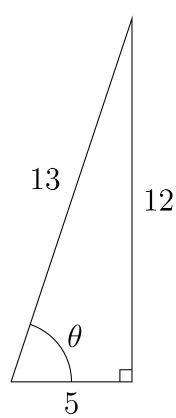
Solution: Recall that $\cos(\pi/6) = \sqrt{3}/2$, so $\arccos(\sqrt{3}/2) = \pi/6$.

3. $\sin^{-1}(\sqrt{3}/2) = ?$

Solution: Recall that $\sin(\pi/3) = \sqrt{3}/2$, so $\sin^{-1}(\sqrt{3}/2) = \pi/3$

4. Draw the right triangle and use it to find the value of $\cos(\sin^{-1}(12/13))$.

Solution: If we let $\sin^{-1}(12/13) = \theta$, then 12 should be opposite of θ, and 13 should be the hypotenuse. Then we can use the Pythagorean Theorem to fill in the last side, so the adjacent side will be 5. Then $\cos(\theta) = 5/13$.



5. $\tan(\arccos(-\sqrt{2}/2)) =?$

Solution: Recall that $\cos(\pi/4) = \sqrt{2}/2$. But here, our ratio is negative, so we need to figure out what θ gives us that $\cos(\theta) = -\sqrt{2}/2$. Note that cos is negative where x is negative, that is in the second and third quadrants. If we reflect the triangle with angle $\pi/4$ into each of these quadrants, we get $3\pi/4$ and $5\pi/4$ respectively. BUT recall that the range of $\cos^{-1}$ is $[0, \pi]$. Since $5\pi/4 > \pi$, our angle can't be that. Thus we have $\arccos(-\sqrt{2}/2) = 3\pi/4$. The problem is actually asking us for the tangent of that, so $\tan(3\pi/4) = -1$.

6. $\cos^{-1}(\sin(\pi/2)) =?$

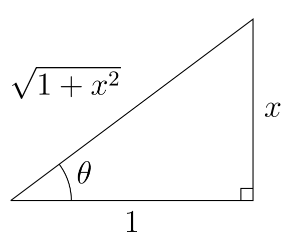
Solution: Recall that $\sin(\pi/2) = 1$, so we're actually looking for $\cos^{-1}(1)$. Since $\cos(0) = 1$, then $\cos^{-1}(1) = 0$.

7. $\arctan(\cos(0)) =?$

Solution: Since $\cos(0) = 1$, we're actually looking for $\arctan(1)$. This means that we're looking for an angle θ so that $\tan(\theta) = 1$. Since $\tan = \frac{\sin}{\cos}$, then $\frac{\sin(\theta)}{\cos(\theta)} = 1$, so $\sin(\theta) = \cos(\theta)$. The only angle that fits that description is $\pi/4$.

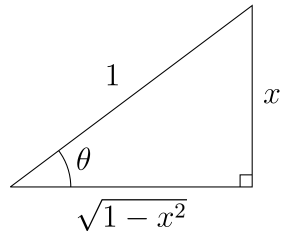
8. Draw the right triangle and use it to find the value of $\sin(\tan^{-1}(x))$.

Solution: Let $\tan^{-1}(x) = \theta$. Then I know that x is opposite of θ and 1 is adjacent to it, so let's solve for the hypotenuse. By the Pythagorean Theorem, it will be $\sqrt{1 + x^2}$. Then sin is opposite over hypotenuse, so this gives us $\frac{x}{\sqrt{1+x^2}}$.



9. Draw the right triangle and use it to find the value of $\cos(\sin^{-1}(x))$.

Solution: Let $\sin^{-1}(x) = \theta$. Then I know that x is opposite of θ and 1 is the hypotenuse, so let's solve for the adjacent side. By the Pythagorean Theorem, it will be $\sqrt{1 - x^2}$. Then cos is adjacent over hypotenuse, so this gives us $\sqrt{1 - x^2}$.



10. Draw the right triangle and use it to find the value of $\sin(\arccos(x))$.

    Solution: Solution: Let $\arccos(x) = \theta$. Then I know that x is adjacent to θ and 1 is the hypotenuse, so let's solve for the opposite side. By the Pythagorean Theorem, it will be $\sqrt{1-x^2}$. Then sin is opposite over hypotenuse, so this gives us $\sqrt{1-x^2}$.